Math 10A
Worksheet, Discussion #19; Thursday, 7/12/2018
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1 Numerical Integration

1. True **FALSE** Using the left endpoint/right endpoint/midpoint rule/trapezoid rule/Simpson's rule to approximate an integral will only give you an approximate answer and never the real answer.

Solution: These methods try to approximate the integral of a function using the area of things that we know. The left/right/midpoint methods try to approximate the function with rectangles and so if your function looks like a rectangle (constant function), then the methods will give you the exact answer. If sections of your function looks like a trapezoid (e.g. a linear function), then the trapezoid method will give you an exact answer. And finally, if your function is already a parabola, then Simpson's method will give you the exact answer since it approximates your function with a polynomial.

2. Approximate $\int_1^2 x^2 dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution: First we divide up our interval $[1, 2]$ into 6 intervals, each of length $\frac{1}{6}$. They are $[1, \frac{7}{6}], [\frac{7}{6}, \frac{8}{6}], [\frac{8}{6}, \frac{9}{6}], [\frac{9}{6}, \frac{10}{6}], [\frac{10}{6}, \frac{11}{6}], [\frac{11}{6}, 2]$. For midpoint, we plug in the midpoint of each interval into our function. Let $f(x) = x^2$, then we have that

$$M_6 = \frac{1}{6}\left[f\left(\frac{1+\frac{7}{6}}{2}\right) + f\left(\frac{\frac{7}{6}+\frac{8}{6}}{2}\right) + \cdots + f\left(\frac{\frac{11}{6}+2}{2}\right)\right].$$

For the trapezoid method, we plug in our function in at the left and right endpoints and take the average of those values, so we have that

$$T_6 = \frac{1}{6}\left[\frac{f(1) + f\left(\frac{7}{6}\right)}{2} + \frac{f\left(\frac{7}{6}\right) + f\left(\frac{8}{6}\right)}{2} + \cdots + \frac{f\left(\frac{11}{6}\right) + f(2)}{2}\right]$$

.

Finally, Simpson's method is a big harder. We need to divide the whole expression by 3 and then the weight which we give each height is in the pattern $1, 4, 2, 4, 2, 4, \ldots, 4, 2, 4, 1$. Doing this gives us

$$S_6 = \frac{1}{3}\cdot\frac{1}{6}\left[f(1) + 4f\left(\frac{7}{6}\right) + 2f\left(\frac{8}{6}\right) + 4f\left(\frac{9}{6}\right) + 2f\left(\frac{10}{6}\right) + 4f\left(\frac{11}{6}\right) + f(2)\right].$$

The results are shown in the table below:

n	L_n	R_n	M_n	T_n	S_n
4	1.968750	2.718750	2.328125	2.343750	2.333333
6	2.087963	2.587963	2.331019	2.337963	2.333333
10	2.185000	2.485000	2.332500	2.335000	2.333333

3. Approximate $\int_0^1 \cos(2x)dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution:

n	L_n	R_n	M_n	T_n	S_n
4	0.622156	0.268119	0.459419	0.445137	0.454811
6	0.568443	0.332419	0.456760	0.450431	0.454680
10	0.523940	0.382325	0.455407	0.453132	0.454653

4. Approximate $\int_0^2 e^{2x}dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution:

n	L_n	R_n	M_n	T_n	S_n
4	15.596437	42.395512	25.714178	28.995975	26.931923
6	18.851333	36.717383	26.309155	27.784358	26.826998
10	21.795632	32.515262	26.621245	27.155447	26.802815

5. Approximate $\int_{-1}^1 x^3dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution:

n	L_n	R_n	M_n	T_n	S_n
4	-0.500000	0.500000	0.000000	0.000000	0.000000
6	-0.333333	0.333333	0.000000	0.000000	0.000000
10	-0.200000	0.200000	0.000000	0.000000	0.000000

6. Approximate $\int_1^3 \ln x dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution:

n	L_n	R_n	M_n	T_n	S_n
4	1.007452	1.556758	1.302645	1.282105	1.295322
6	1.106594	1.472798	1.298895	1.289696	1.295721
10	1.183758	1.403480	1.296944	1.293619	1.295821

7. Approximate $\int_1^2 xe^x dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution:

n	L_n	R_n	M_n	T_n	S_n
4	5.968575	8.983532	7.345610	7.476053	7.389616
6	6.422771	8.432742	7.369716	7.427756	7.389167
10	6.800003	8.005986	7.382088	7.402995	7.389071

8. Approximate $\int_1^4 \sqrt{x} dx$ using the midpoint rule, trapezoid rule, and Simpson's rule with $n = 4$.

Solution:

n	L_n	R_n	M_n	T_n	S_n
4	4.280093	5.030093	4.672401	4.655093	4.666221
6	4.411488	4.911488	4.669245	4.661488	4.666563
10	4.514796	4.814796	4.667601	4.664796	4.666652